

# (Non-) geodesic motion in chameleon Brans Dicke model

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Based on [16], we assume there is a non-minimal coupling between the scalar field and matter in Brans-Dicke model. We analyze the motion of different matter such as, massless scalar field, photon, massless perfect fluid (dust), massive perfect fluid and point particle matter in this theory. We show that the motion of massless scalar field and photon can satisfy null geodesic motion only in high frequency limit. Also we find that the motion of the dust and massive perfect fluid is geodesic for  $L_m = -P$  and is non-geodesic for  $L_m = \rho$ . Finally, we study the motion of point particle and show that the motion of this kind of matter is non-geodesic.

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## I. INTRODUCTION

Contemporary cosmology is encountered with the important challenge of understanding the existence and nature of the dark energy component of the Universe. Analysis of cosmological observations suggests that about 74% of the Universe is dark energy (DE), 22% is dark matter (DM) and the remaining part is ordinary matter [1]. Although the nature and origin of DE are unknown for researchers until now, there are many proposals to explain the role of DE to explain the accelerating expansion of the Universe. It seems that the best one is cosmological constant,  $\Lambda$ , which has the equation of state (EoS) parameter  $\omega = -1$ , [2–4], and the second popular candidate of DE model are the scalar field models with a dynamical equation of state. The most important dynamical DE model which is called “quintessence” model consider the slow-roll down of a scalar field. Although, the quintessence scalar field cannot satisfy the local tests (solar system constraints), but it suggests an energy form with negative pressure to explain the accelerating expansion of the Universe [5–12].

Another suitable framework to investigate the behavior of DE is chameleon mechanism. In this mechanism the scalar field has non-minimal coupling with matter. Chameleon mechanism provides an alternative mechanism for circumventing the constraints from local tests of gravity. In this mechanism the scalar field acquires a mass whose magnitude depends on the local matter density. Indeed this mechanism is a way to give an effective mass to a light scalar field via field self interaction and interaction between field and matter [13, 14]. So because of this fact the correction of physical quantity in Newtonian regime is small, i. e., the local tests are satisfied.

Another model that has attracted much attention is Brans-Dicke (BD) theory. Although BD theory proved

useful for solution of many cosmological problems, but it has a problem. Indeed, the Brans-Dicke parameter,  $\omega$ , takes small value ( $\sim 1$ ) when standard BD model is used to derive the cosmic acceleration, and on the other hand, local constraints require that  $\omega > 10^4$ . So, some researchers such as, Clifton *et al.*, [15] and Das *et al.*, [16], have studied another framework which scalar field has non-minimal coupling with both geometry and matter. This model is called chameleon Brans-Dicke (CBD) model and it has predicated a value for  $\omega$  which is in a good agreement with observational data [17].

As mentioned earlier, there are a lot of attempts to explain the positive accelerating expansion of the Universe. In order to do that, people have introduced various models for example; CBD model. Actually CBD model is studied in detailed on large scale, solar system scale and it is shown that the obtained results are in a good agreement with observations [18–23].

However when a scalar field interacts with other components of matter (visible matter and invisible matter [37]) through gravity or directly, this interaction may produce a fifth force on the matter which may violate the weak equivalence principle (WEP) and creates a non-geodesic motion. This kind of interactions have attracted much attention [24]. There are some particular mechanism for circumventing the fifth force effects. Some researchers believe that the scalar field coupled differently to visible and invisible matter of the Universe [25–27]. Therefore based on this opinion, for suppressing the effects of fifth force, they assume that the scalar field couples only to the invisible matter [26]. Another mechanism for circumventing the fifth force and then the violation of the WEP, is chameleon mechanism. As was mentioned earlier, the mass of chameleon scalar field is a function of local density and in the high density regions [38] the fifth force effects are confined to an undetectable small distances. Therefore the violation of WEP is not observed. The WEP violation had become the hot topic and studied in detail [13, 28]. But there are still another aspects of CBD model which should be studied and to the best of our knowledge, there is no any detailed study about

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geodesic and non-geodesic motion for different kinds of matter in CBD model.

This paper is organized as follows. In Sec. II, we consider the model, then we will obtain the equations of motion and conservation relation for density energy. In Sec. III, we will study the motion of massless scalar field and photon. In Sec. IV we consider the motion of perfect fluid and point particle in this model. At last we summarize our work and give some discussion in Sec. V.

Throughout this paper, the metric signature  $(-, +, +, +)$  and the convention  $8\pi G = 1$  are used.

## II. GENERAL FRAMEWORK

We begin with the chameleon Brans-Dicke action [16]

$$A = \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} \nabla_a \phi \nabla^a \phi - V(\phi) - 2f(\phi) L_m \right], \quad (1)$$

where  $R$  is the Ricci scalar curvature,  $g$  is determinant of metric,  $\omega$  is the dimensionless CBD parameter,  $L_m = L_m(\psi, g_{ab})$  is the Lagrangian of the matter and  $\psi$  is the matter field.  $\phi$  is the CBD scalar field with a potential  $V(\phi)$ . Note that the last term in the action indicates the interaction between the matter and an arbitrary function of scalar field,  $f(\phi)$ .

The gravitational field equation can be derived by taking account variation of (1) with respect to  $g_{ab}$  and is given by

$$\phi G_{ab} + [g_{ab} \square - \nabla_a \nabla_b] \phi = f(\phi) T_{ab} + T_{ab}^\phi, \quad (2)$$

where

$$T_{ab} = 2\phi \sqrt{-g} \delta(\sqrt{-g} L_m) \delta g^{ab}, \quad (3)$$

is the the matter energy-momentum tensor and

$$T_{ab}^\phi = \omega \phi \left[ \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi \right] - \phi \nabla_a \nabla_b V(\phi), \quad (4)$$

is the scalar field energy-momentum tensor. Also by taking the variation of (1) with respect to  $\phi$  we have the equation of motion for scalar field

$$(3 + 2\omega) \square \phi = f(\phi) T - \phi f'(\phi) L_m + \phi V'(\phi) - 2V(\phi). \quad (5)$$

Here  $T$  is the trace of matter energy-momentum tensor and prime denotes derivative with respect to  $\phi$ . It is seen that to solve (5) we need an explicit form of matter Lagrangian,  $L_m$ .

The Bianchi identities, together with the identity  $(\square \nabla_a - \nabla_a \square) V_c = R_{ab} \nabla^b V_c$ , imply the non-(covariant) conservation law

$$\nabla_a T^{ab} = -[g^{ab} L_m + T^{ab}] \nabla_a \ln(f), \quad (6)$$

and, as expected, in the limit  $f(\phi) = \text{constant}$ , one recovers the conservation law  $\nabla_a T^{ab} = 0$ . Since the energy-momentum tensor is not covariantly conserved, one may

concludes that the motion of matter distribution characterized by a Lagrangian density  $L_m$  is nongeodesic. This fact that the energy-momentum tensor is not divergence-free, can be interpreted as a violation of the so-called metric postulates [29].

## III. MATTER-SCALAR COUPLING AND GEODESICS OF MASSLESS MATTER FIELD

As was mentioned earlier in the Introduction, the explicit coupling between matter and scalar field which is described by the action (1) can potentially lead to non-geodesic motion. In this section we consider the affect of this kind of interaction on massless particles geodesics.

### A. Massless scalar field

Let us consider a massless scalar field  $\psi$  which is described by the Lagrangian density

$$L_m = -\frac{1}{2} \nabla_a \psi \nabla^a \psi. \quad (7)$$

Using

$$T_{ab} = \partial L / \partial (\nabla^a \psi) \nabla_b \psi - g_{ab} L,$$

one can obtain the stress-energy tensor of the scalar field

$$T_{ab} = -\nabla_a \psi \nabla_b \psi + \frac{1}{2} g_{ab} \nabla_c \psi \nabla^c \psi, \quad (8)$$

Substituting (7) and (8) in (6) we have

$$\square \psi = -\nabla_a \psi \nabla^a \ln(f), \quad (9)$$

One can see that for  $L_m$  and  $L_m + \nabla_c \chi$  ( $\chi$  is a scalar function), Eq. (9) is not changed just for  $\partial \chi_{,c} / \partial \nabla^a \psi = \nabla_a \chi_{,c} = 0$ , but equation of motions for other components of the system, Eqs. (5) and (6), are changed in this case. This means that when massless scalar matter couple with other components of the system, there is no any degeneracy of Lagrangian densities.

Note that this kind of matter is known as "scalar photon". So, although our study is completely classic, but we assume this scalar photon has a wave like behavior, then the matter scalar field,  $\psi$ , can be as a wave function. Therefore we assume the wave function is a high frequency wave as

$$\psi(x) = \psi_0 e^{i\Phi(x)}. \quad (10)$$

Here the phase of wave is a rapidly varying function of  $x$  and  $\psi_0$  is nearly constant. Therefore Eq.(8) becomes

$$i \square \Phi(x) - \nabla^a \Phi \nabla_a \Phi = -i \nabla_a \Phi \nabla^a \ln(f). \quad (11)$$

Since this equation has tow real and imaginary parts, we have

$$\nabla^a \Phi \nabla_a \Phi = 0, \quad (12)$$

$$\square \Phi(x) = -\nabla_a \Phi \nabla^a \ln(f), \quad (13)$$

In comparison with similar analysis in standard model of cosmology, Eq. (??) shows that the scalar particle is not transverse unless  $\nabla_a \Phi$  will be orthogonal to  $\nabla^a \ln(f)$  or  $\nabla^a \ln(f) = 0$ . Also by taking covariant derivative of Eq. (12) and using  $\nabla_a \nabla_b \Phi = \nabla_b \nabla_a \Phi$ , one can obtain

$$\nabla_v v_a = 0, \quad (14)$$

where  $v_a = \nabla_a \Phi$ . Note that  $v_a = \nabla_a \Phi$  is the gradient of the wave phase and in the geometric optic approximation, this quantity is the tangent of the worldline of the particle (massless scalar particle). Therefore in comparison to real photon, Eq. (14) means that the motion of massless scalar particle in the high frequency limit is take place on the null geodesic.

### B. Maxwell field

Let us consider the Maxwell field with Lagrangian density and energy- momentum tensor

$$L_m = -1\phi 16\pi F^2, \quad (15)$$

$$T_{ab} = -1\phi 4\pi \left[ F_{ac} F_b^c - 1\phi 4g_{ab} F^2 \right], \quad (16)$$

where  $F_{ab}$  is the electromagnetism field tensor on the curve space-time

$$F_{ab} = \nabla_a A_b - \nabla_b A_a = \partial_a A_b - \partial_b A_a, \quad (17)$$

and  $A_a$  is the vector potential. Substituting (15) and (16) into (6) gives

$$\nabla^b \left[ F_{ac} F_b^c - 1\phi 4g_{ab} F^2 \right] = -F_{ac} F_b^c \nabla^b (\ln f). \quad (18)$$

Now we consider the high frequency limit. For this goal we introduce the vector potential as

$$A_a(x) = C_a e^{i\Phi(x)} \quad (19)$$

where  $C_a$  is a slowly varying vector amplitude (nearly constant) and  $\Phi(x)$ , the phase of wave function, is a rapidly varying function. So by neglecting the derivatives of vector amplitudes,  $C_a$ , we have

$$2\mathcal{A}_{ab} \nabla^b \Phi - \nabla_a \Phi \left[ (\nabla \Phi)^2 C^2 - (\nabla_c \Phi C^c)^2 \right] = 0, \quad (20)$$

$$2\nabla^b \left[ \mathcal{A}_{ab} f(\phi) \right] - \nabla_a \left[ (\nabla \Phi)^2 C^2 - (\nabla_c \Phi C^c)^2 \right] = 0, \quad (21)$$

where

$$\begin{aligned} \mathcal{A}_{ab} = & C^2 \nabla_a \Phi \nabla_b \Phi + (\nabla \Phi)^2 C_a C_b \\ & - (C^c \nabla_c \Phi) \left[ C_b \nabla_a \Phi + C_a \nabla_b \Phi \right], \end{aligned} \quad (22)$$

and  $C^2 = C^c C_c$ ,  $(\nabla \Phi)^2 = \nabla_c \Phi \nabla^c \Phi$ . Using Eqs. (20) and (22), one can find

$$C^2 (\nabla \Phi)^2 = (C^c \nabla_c \Phi)^2, \quad (23)$$

and by substituting (23) into (20) and (21) we have

$$\mathcal{A}_{ab} \nabla^b \Phi = 0, \quad (24)$$

$$\nabla^b \mathcal{A}_{ab} = -\mathcal{A}_{ab} \nabla^b \ln(f). \quad (25)$$

By setting  $f(\phi) = 1$  one can arrive at the standard Maxwell equations in curved space. Geometric optics is valid whenever the wavelength is very short with respect to the radius of curvature of space-time, namely  $\bar{\lambda} \ll \mathcal{L}$ , here  $\mathcal{L}$  is the radius of curvature of space-time, and  $\bar{\lambda}$  is the reduced wave length of photon. One can write Eq. (25) as

$$|\nabla^b \mathcal{A}_{ab}| \simeq |\mathcal{A}_{ab} \phi \bar{\lambda}| \gg |\mathcal{A}_{ab} \ln(f) \phi \mathcal{L}| \quad (26)$$

So, in this case we have

$$\nabla^b \mathcal{A}_{ab} \simeq 0. \quad (27)$$

This shows that the corrections to standard optics which is coming from interaction between scalar field and matter character,  $f(\phi)$ , completely removed from the equation of motion and then photons follow the null geodesics and are transverse. On the other hand for the case that  $\bar{\lambda}/\mathcal{L} \not\ll 1$ , one can not disappear the non-minimal coupling affect to the Maxwell equations and then the null geodesic equation of photon is modified.

### IV. MATTER-SCALAR FIELD COUPLING AND GEODESICS OF PERFECT FLUID MATTER

In this section we consider a kind of matter, so called perfect fluid, which can be massive or massless. The stress-energy tensor of perfect fluid is represented by

$$T_{ab} = (\rho + P) U_a U_b + g_{ab} P, \quad (28)$$

where  $\rho$  is the energy density and  $P$  is the pressure of the matter respectively, and the four velocity,  $U_a$  satisfies the constraints  $U_a U^a = -1$  and  $U^a \nabla_b U_a = 0$ . In [30–34] have been shown that, for perfect fluid that does not couple explicitly to the other components of the system, there are different Lagrangian densities which are perfectly equivalent. In fact, they have shown that, by using Eq. (3), the two Lagrangian densities  $L_{m_1} = -P$  and  $L_{m_2} = \rho$  give the same stress-energy tensor as (28), and also for these two different Lagrangian densities the equation of motions for all components of the system is similar. Also, since the perfect fluid laws are obtained via a kinetic theory by using microscopic models of the fluid particles and their interaction, namely, the perfect fluid is an averaged and not an exact description for matter, it is more common to work directly with the energy-momentum tensor instead of Lagrangian density in the non-interacting

model of perfect fluid. But in our model the Lagrangian density,  $L_m$ , is explicitly appeared in equation of motion of scalar field, (5), and conservation relation, (6). So we encounter with a new situation that we have to study it accurately.

We can work with stress-energy tensor of perfect fluid only for the case which there is no any interaction between perfect fluid and other components of the system. This means that if there is a direct interaction between perfect fluid and other components of system, such as geometry and scalar field, the above Lagrangian densities give rise to distinct theories with different predictions. To show this fact, let us consider a general case.

We assume there is a minimal coupling between perfect fluid and scalar field, i.e.,  $f(\phi) = 1$ . In this case the Lagrangian of perfect fluid is not appeared explicitly in the equation of motions of other components of the system and the energy-momentum tensor of matter is conserved, namely

$$\begin{aligned} \phi G_{ab} + [g_{ab}\square - \nabla_a \nabla_b]\phi &= T_{ab} + T_{ab}^\phi, \quad (29) \\ (3 + 2\omega)\square\phi - \phi V'(\phi) + 2V(\phi) &= T, \quad (30) \\ \nabla_a T^{ab} &= 0, \quad (31) \end{aligned}$$

where matter energy-momentum tensor,  $T_{ab}$ , and the scalar field energy-momentum tensor,  $T_{ab}^\phi$ , are given by Eqs. (3) and (4). Also we suppose there are two different Lagrangian density  $L_{m_1}$  and  $L_{m_2}$  which by using (3) give an energy-momentum tensor like (28), and  $L_{m_1}$  and  $L_{m_2}$  are related with together by

$$L_{m_2} = L_{m_1} + \frac{1}{\sqrt{-g}} \nabla_a \chi \quad (32)$$

where  $\chi$  is a scalar function. Since the perfect fluid Lagrangian is not appear in equation of motions, (29) and (30) and conservation relation of energy, (31), then these two Lagrangian has not any effect on equation of motion of other components of the system and conservation relation of energy. Also the additional term,  $\nabla_a \chi / \sqrt{-g}$ , in Eq. (1) give a surface integral term, then this term has no any effect on equation of motions of perfect fluid. Therefore these two different Lagrangian are equivalent for perfect fluid in a non-interacting model.

On the other hand we assume, there is an interaction between perfect fluid and other components of system, i.e., for an arbitrary  $f(\phi)$ . It is obviously seen that the equation of motion of perfect fluid for two different Lagrangian densities, (32), are similar, moreover, by using Eq. (3) one can obtain a matter energy-momentum tensor as (28). But the equation of motion of scalar field and also the conservation relation of energy for matter become equations (6) and (7) respectively, which are different with Eqs. (29) and (30) and they are different for  $L_{m_1}$  and  $L_{m_2}$ . This fact shows that, clearly, the two Lagrangian density  $L_{m_1}$  and  $L_{m_2}$  cannot be equivalent in an interacting system of perfect fluid with other components of the system.

### A. Null geodesic of dust

The equation of null geodesics for a model with out any coupling is derived from the conservation equation of a null dust fluid in [35]. Therefore, in this case which there is a coupling between scalar field and matter we do the same way of derivation. If this interaction were to induce any corrections to the null geodesic equation, these has to show up in this derivation. Since dust is a perfect fluid without pressure, namely  $P = 0$  then the stress-energy is  $T_{ab} = \rho U_a U_b$ , so by using the modified conservation equation, (7), we have

$$\nabla_u U_a = \eta U_a, \quad (33)$$

where  $\nabla_u = U^b \nabla_b$  and

$$\eta = -\nabla_u \ln(f\rho) - \nabla^b U_b, \quad (34)$$

Eq. (33) is a geodesic equation which is non-affinely parameterized. In fact this equation shows that the four velocity is transported along the path parallel to itself and this is the definition of a geodesic curve. So this means that the existence of coupling between the matter and scalar field does not change the equation of null geodesic for  $L_m = -P$ . But for  $L_m = \rho$  we obtain

$$\nabla_u U_a = \eta U_a - \nabla_a \ln(f), \quad (35)$$

where clearly shows that parallel transport is no longer conserved and then the motion of dust particle is non-geodesic in this case.

### B. Massive perfect fluid matter

In this section we turn our attention to massive matter fields and, for simplicity, consider a perfect fluid composed of non-relativistic or relativistic particles with stress-energy tensor (28). This stress-energy tensor is obtained from (3) with  $L_m = -P$ . By defining a projection operator as  $h_{ab} = g_{ab} + U_a U_b$ , one can project equation (6) onto the direction normal to the four velocity as

$$\nabla_b T^{ab} = -[g_{ab} L_m + T_{ab}] \nabla^b (\ln f). \quad (36)$$

Using (28), one can obtain the non-geodesic motion for the fluid element as

$$\nabla_u U^c = f^c, \quad (37)$$

where the extra force,  $f^c$  is given by

$$f^c = -1/\phi(P + \rho) \left[ (L_m + P) \nabla_b (\ln f) + \nabla_b P \right] h^{ac}. \quad (38)$$

So by using  $L_m = -P$  we have

$$f^c = -1/\phi(P + \rho) \nabla^c P. \quad (39)$$

This states the extra force is related to the coupling between matter and scalar field and it is proportional to

the pressure gradient. This term is the usual term that appears in standard GR and encapsulates the force exerted on a fluid element due to the fluid pressure. This means that energy is indeed not conserved for this fluid does not affect the geodesic motion in GR.

Also by substituting (28) in to (37) one can obtain

$$\nabla_a T^{ab} = -(P + \rho)[\nabla_u(\ln f)]U^b, \quad (40)$$

this equation states the flow of energy only take places along the direction of  $U$  i.e., aligned with the fluid world-lines. This means that the spatial components of the force in the rest frame of the fluid is zero and only the time component of the force is nonzero. This kind of force cannot have any effect on the motion because, based on the normalization  $U^a U_a = -1$ , the four acceleration  $a_c$  is perpendicular to the four velocity  $U^c$ . This states that the components of four-force perpendicular to the four velocity is zero. This is the fact which we discover here. Note that according to (38) in the case of dust with  $P = 0$ , the extra force  $f^c$  is zero and this is agree with (33).

On the other hand by inserting the Lagrangian density  $L_m = \rho$  in Eqs. (36) and (37) we have

$$\nabla_a T^{ab} = -(P + \rho)[\nabla^b(\ln f)], \quad (41)$$

$$f^c = -1\phi(P + \rho)\nabla^c P - \nabla^c \ln(f). \quad (42)$$

Equation (41) states the flow of energy is not along the direction of  $U$ . This means that all components ( spatial and time components ) of the fifth force in the rest frame of the fluid are nonzero and then this kind of force can have any effect on the motion of matter. Also Eq. (42) shows that the fifth force does not proportional to the pressure gradient and it depends to the coupling between matter and scalar field. This means that in this case the motion is not geodesic.

### C. Massive matter

In this Subsection we want to study the motion of ordinary massive matter (not perfect fluid). For this goal, we begin with energy-momentum four-vector  $p_0^a(t)$ . The density of  $p_0^a(t)$  is defined by [36]

$$T^{a0}(x, t) = p_0^a(t)\delta(\mathbf{x} - \mathbf{x}_0), \quad (43)$$

where  $\mathbf{x}$  is the general coordinate,  $\mathbf{x}_0$  is the coordinate of center of particle ( the index "0" indicates the center of particle). And the current of this four vector is defined by

$$T^{ai}(x, t) = p_0^a(t)\frac{dx_0^i(t)}{dt}\delta(\mathbf{x} - \mathbf{x}_0), \quad (44)$$

one can united these two definition into one as

$$T^{ab}(x, t) = p_0^a(t)\frac{dx_0^b(t)}{dt}\delta(\mathbf{x} - \mathbf{x}_0), \quad (45)$$

where  $x_0^0(t) := t$ . By rewriting the energy-momentum tensor of particle, (45), in the co-moving coordinate and using  $p_0^a(\tau) = m_0 u^a$ , we have

$$T^{ab}(x) = m_0 u^a u^b, \quad (46)$$

where  $\tau$  is the proper time,  $u^a$  is four velocity vector and  $u^a u_a = -1$ .

Moreover according to [27], we introduce a matter Lagrangian for a point particle with mass  $m_0$  by

$$L_m = m_0 \delta(\mathbf{x} - \mathbf{x}_0) \sqrt{-g_{ab} \dot{x}_0^a \dot{x}_0^b}, \quad (47)$$

where  $\dot{x}_0^a = dx_0^a/d\tau$ . The Lagrangian (47) give the particle equation of motion as

$$\ddot{x}_0^a + \Gamma_{bc}^a \dot{x}_0^b \dot{x}_0^c = 0, \quad (48)$$

This equation is a geodesics equation of motion for point particle. Since  $g_{ab} \dot{x}_0^a \dot{x}_0^b = g_{ab} u^a u^b = -1$ , one can rewrite (47) in co-moving coordinate as

$$L_m = m_0. \quad (49)$$

Substituting Eqs. (46) and (49) into Eq. (6) we get

$$\nabla_a u_b = -\tilde{\nabla}_b \ln(f) - (\nabla_a u^a)u_b. \quad (50)$$

Eq. (50), is geodesic equation of motion for a particle in CBD model which is modified with respect to conventional geodesic equation of GR. There are two terms in the right side of Eq. (50) which are the fifth force contribution from non-minimal coupling between matter and scalar field. Note that  $\tilde{\nabla}_b := (g_{ab} + u_a u_b)\nabla^a$  is a particular derivative in the 3-dimensional space perpendicular to  $u_a$ . Therefore  $\tilde{\nabla}_b \ln(f)$  is perpendicular to  $u_a$ , so it doesn't any effect on the magnitude of velocity, namely this term does not any effects on the energy of particle. Also, since  $(\nabla_a u^a)u_b$  is aligned on the particle world-line, it does not any effects on parallel transport of four-velocity.

## V. CONCLUSION

We have studied Brans-Dicke model which include a non-minimal coupling between scalar field and matter, so-called chameleon Brans-Dicke model. We have considered the possible deviation of free fall trajectories from geodesics. We have studied the motion of massless scalar particles, photon, massless perfect fluid(dust), massive perfect fluid and finally ordinary massive particles.

By assuming a wave like behavior for massless scalar field ( scalar photon), we have shown that the motion of scalar photon is take place on null geodesics only for high frequency limit, Moreover we have found that the electromagnetic particle (photon) is transverse and the motion of it is null geodesics only for the case which the reduced wave length of the photon be very small with

respect to radius of curvature of the space-time and for the case  $\bar{\lambda}/\mathcal{L} \ll 1$  the photon does not transverse and the motion of it is not null geodesic.

Furthermore, we have discussed the (non)- geodesics motion of perfect fluid. We have found that, although there is a degeneracy of Lagrangian densities in the context of standard GR, but our analysis have shown that in the CBD model this degeneracy does not excite and also dust and massive perfect fluid have geodesics motion for  $L_m = -P$  and the motion of them is non-geodesics for  $L_m = \rho$ .

Finally, we introduced a Lagrangian and energy-

momentum tensor for a point like particle, and we have shown that the motion of a massive particle is not geodesic in CBD theory.

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- [37] Invisible matter is not completely cold dark matter because some candidates of cold dark matter are visible.
- [38] Where observation and experiments are performed such as Earth.